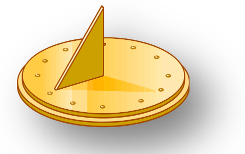
Lesson 4: Introduction to Triangle Trigonometry

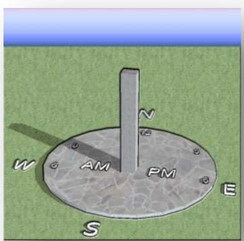
Standard: MCC9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **MCC9-12.G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **MCC9-12.G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Essential Question: What are trigonometric ratios? How do you find the sine, the cosine, and the tangent of an acute angle?

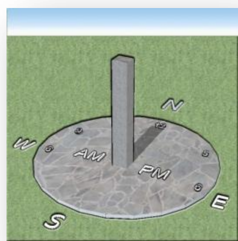


The word ***trigonometry*** is of Greek origin and literally translates to "Triangle Measurements". Some of the earliest trigonometric ratios recorded date back to about 1500 B.C. in Egypt in the form of sundial measurements. They come in a variety of forms. The most basic sundials use a simple rod called a gnomon that simply sticks straight up out of the ground. Time is determined by the direction and length of the shadow created by the gnomon.

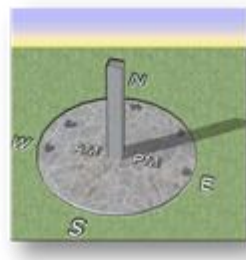
In the morning the sun **rises in the east** and alternately the shadow created by the gnomon points westerly. When the sun reaches its highest point in the sky it is known as '**High Noon**'. At 12:00 p.m. noon the shadow of a gnomon in a simple sundial is at its shortest length and points due north (at least it does so in the northern hemisphere). Then as the sun **sets in the west,** the shadow of the gnomon points east (as shown in the pictures below).



9:00 a.m.



12:00 p.m.

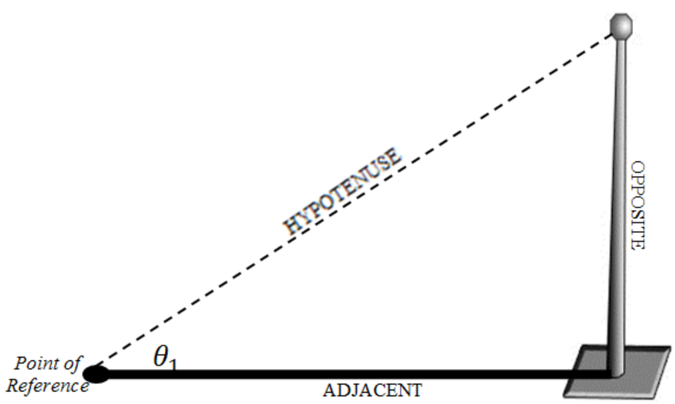


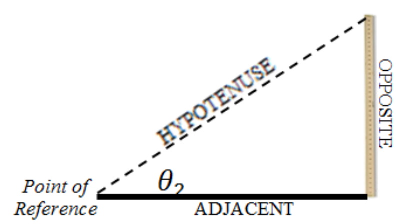
3:30 p.m.

Notice how the shadow rotates throughout the day on the sundial shown. These were the earliest clocks. The shadows acts like the hand of a clock moving in a clockwise motion. This is the reason clock's hands today move in the direction they do today.

By creating a segment from the top of the gnomon to the tip of the shadow a right triangle is formed. Some of the earliest mathematicians charted the placement of the shadows over time and seasons and they began to analyze the relationships of the measurements of the right triangle create by these sundials.

1. Consider the following diagrams of sundials. Let the vertex at the tip of the shadow be the point or angle of reference. Below show two examples of makeshift sundials using a **flagpole** and **meter stick**. Both diagrams represent 7:30 a.m. Using a ruler measure the length of each side of each triangle in the diagrams using centimeters to the nearest tenth.





SINE is the ratio of Opposite to Hypotenuse (abbreviated '**sin**').	$\sin\theta = \dfrac{Length\ of\ Opposite\ Side}{Length\ of\ Hypotenuse}$	***SOH***
COSINE is the ratio of Adjacent to Hypotenuse (abbreviated '**cos**').	$\cos\theta = \dfrac{Length\ of\ Adjacent\ Side}{Length\ of\ Hypotenuse}$	***CAH***
TANGENT is the ratio of Opposite to Adjacent (abbreviated '**tan**').	$\tan\theta = \dfrac{Length\ of\ Opposite\ Side}{Length\ of\ Adjacent\ Side}$	***TOA***

2. Fill in the charts below with the measurements from problem #1. The ratios of the sides of right triangles have specific names that are used frequently in the study of trigonometry.

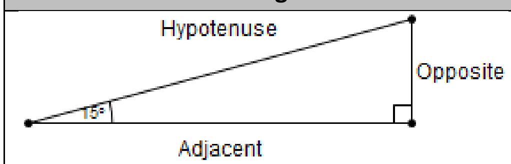
Flag Pole Triangle	
Opposite	
Adjacent	
Hypotenuse	
$\sin\theta_1 = \dfrac{Opposite}{Hypotenuse}$	
$\cos\theta_1 = \dfrac{Adjacent}{Hypoentuse}$	
$\tan\theta_1 = \dfrac{Opposite}{Adjacent}$	
θ_1 (using protractor)	

Meter Stick Triangle	
Opposite	
Adjacent	
Hypotenuse	
$\sin\theta_2 = \dfrac{Opposite}{Hypotenuse}$	
$\cos\theta_2 = \dfrac{Adjacent}{Hypoentuse}$	
$\tan\theta_2 = \dfrac{Opposite}{Adjacent}$	
θ_2 (using protractor)	

3. What do you notice about the two tables? Can you suggest any reasons for your conclusion?

Using what you noticed in problem #3, some of the earliest mathematicians carefully collected approximate table of ratios for varying reference angles.

4. Below are a variety of triangles. Measure each side in centimeters to the nearest tenth.

<table>
<tr><th>Triangle</th><th>Opp.</th><th>Adj.</th><th>Hyp.</th><th>sin θ</th><th>cos θ</th><th>tan θ</th></tr>
<tr><td>
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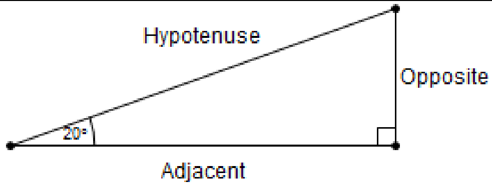
<tr><td>
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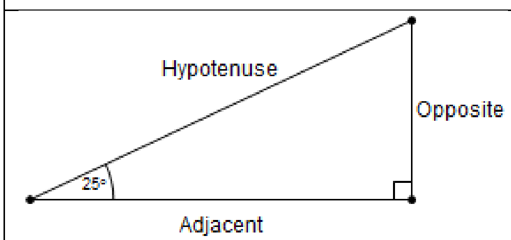
<tr><td>
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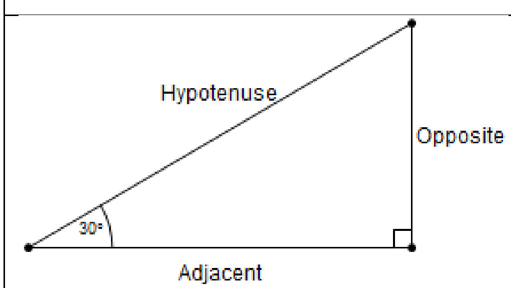
<tr><td>
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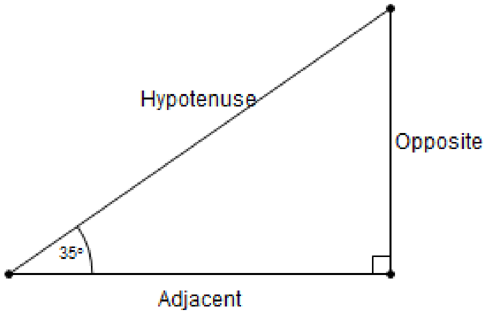
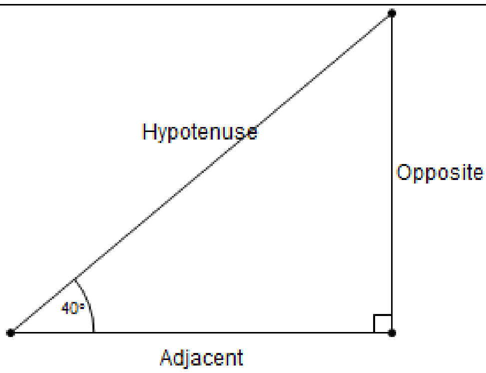
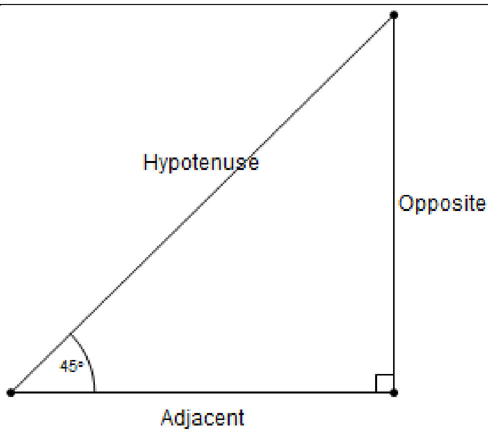
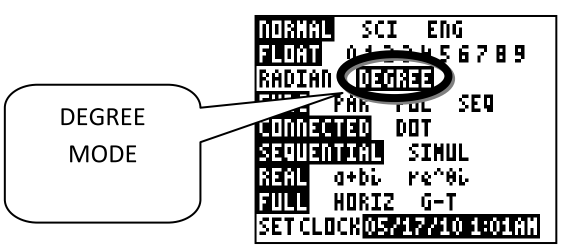
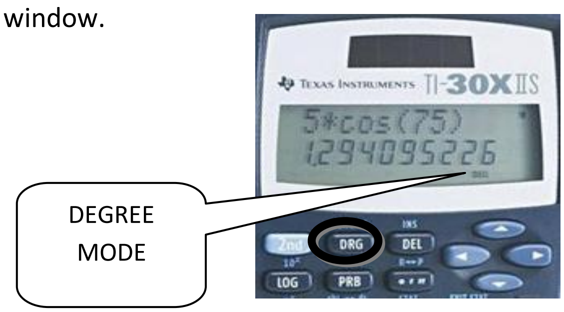
						
						
						

Your calculator can approximate these ratios. First you'll need to make certain your calculator is in DEGREE mode (there are multiple ways to measure angles and we are currently using degrees).

TI-83/84: Press the  button. Then, use the arrow keys to highlight DEGREE and press  .

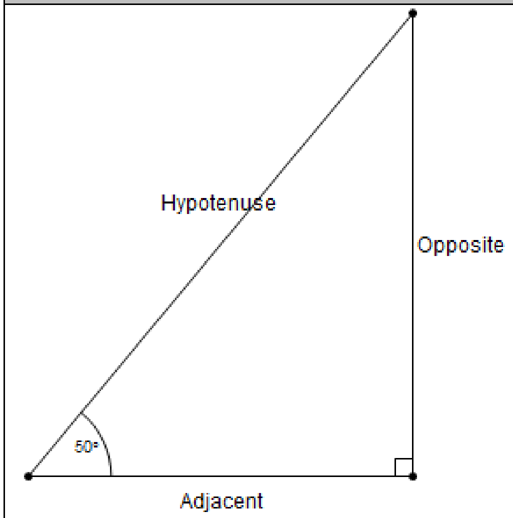
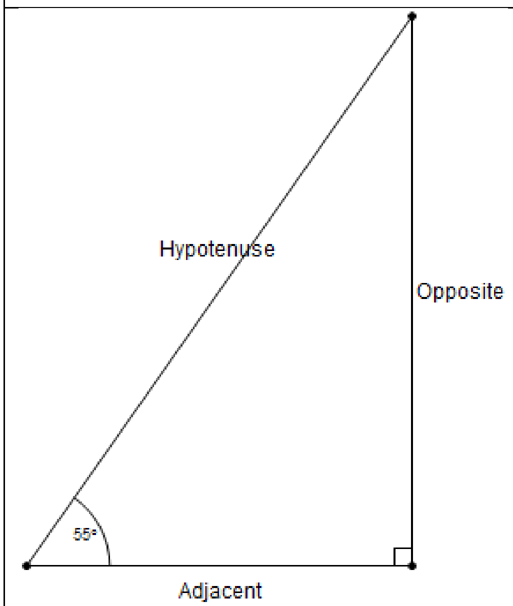


TI-30II: Press  until "DEG" shows up in the calculator window.



Using the calculator check a couple of your ratios in the table above.

4. (continued) Below are a variety of triangles. Measure each side in centimeters to the nearest tenth.

Triangle	Opp.	Adj.	Hyp.	sin θ	cos θ	tan θ
						
						

5. 40° and 50° are **complementary** angles because they have a sum of 90°.

a. What is an approximation of sin (40°)?

b. What is an approximation of cos (50°)?

c. What is an approximation of sin (30°)?

d. What is an approximation of cos (60°)?

e. What is an approximation of sin (55°)?

f. What is an approximation of cos (25°)?

g. What do you think the "CO" in **CO**SINE stands for